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GIVING ACCORDING TO AGREEMENT

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Giving According To Agreement

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Abstract

We propose an axiom that we call Agreement to deal with changing preferences and derive its empirical implications. The resulting revealed preference condition generalises GARP when preferences are different but preferences in one context are informative about preferences in another context. We apply this idea to a social choice experiment, where a player can respond to another player being kind or relatively unkind. We find that people have a consistent preferences for each case, but that preferences depend on the kindness of the other player, and that subjects act in line with Agreement. We thus provide support for modelling and interpreting responses to the intentions of other players as a preference for reciprocity.

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1 Introduction

Many economic decisions are made in a social context. Transactions often involve repeated interactions with owners, managers, employees, suppliers, service providers, or consumers. In such situations, reciprocity can play an important role. Reciprocity refers to a tendency of responding in kind to behaviour or the perceived intention behind that behaviour. This can sustain cooperation or conflict where this would not occur otherwise. It introduces path dependency into choice settings: the evaluation of an alternative will depend on how pleasant or unpleasant previous interactions were.

Considerable evidence has been found for reciprocal behaviour. [Blount \(1995\)](#) and [Falk et al. \(2008\)](#) find with experimental data that subjects give less to or even reduce the payoff of another player if this player rather than a randomisation device gives them a low endowment. [Falk et al. \(2003\)](#) find that in an ultimatum game, the same offer is more likely to be rejected if more equitable allocations could have been chosen by the proposer than if the proposer could only have offered less equitable allocations. In a lab experiment, [Charness \(2004\)](#) finds that employees exert more effort when they are paid a higher wage by an employer, and that they exert particularly low effort if a low wage is set by the employer rather than by a randomisation device or the experimenter. In a field experiment, [Cohn et al. \(2015\)](#) similarly find that paying a higher wage increased performance of workers performing a one-time job. [Dohmen et al. \(2009\)](#) find evidence for reciprocity in a representative survey, and find that, in line with experimental findings, reciprocity is correlated with wages and employment. [Falk et al. \(2018\)](#) find evidence of reciprocity in a large representative sample of 76 countries, and find that it is correlated with the frequency of armed conflict.

Findings of reciprocal behaviour are often interpreted and modelled as reflecting a preference, an approach [Sobel \(2005\)](#) calls *intrinsic reciprocity*. Whether or not reciprocity is a preference has important implications. If reciprocity is intrinsic, we can use the revealed preference methodology or the utility maximisation paradigm to learn about reciprocal behaviour. It has major implications for welfare analysis: if preferences are reciprocal, i.e. depend on previous interactions (apart from their effect on final allocations), then people essentially have different preference relations depending on the kindness of

the other player. This complicates welfare analysis, as it is not obvious according to which preference relation we should judge welfare. A method such as the one proposed by [Bernheim and Rangel \(2007\)](#) must then be used for welfare analysis.

Previous modelling efforts (e.g. [Rabin 1993](#); [Dufwenberg and Kirchsteiger 2004](#); [Falk and Fischbacher 2006](#); [Segal and Sobel 2007](#)) have focused mostly on the game-theoretical implications of reciprocity, rather than on measuring and comparing preferences. As a consequence, they make specific functional form assumptions about important unobservables such as the utility function and notions of fairness. Tests of these models are simultaneously tests of their parametric assumptions. Because the assumptions are made for tractability, not realism, they are likely to lead to violations even if people have reciprocal preferences. As [Sobel \(2005, p. 395\)](#) writes, “Rejecting a model is easy, rejecting an approach is nearly impossible.”

In this paper, we focus on the simplest hypothesis generated by the notion of intrinsic reciprocity for which it is possible to design a test with the power to reject the hypothesis: if reciprocity is a preference, there should exist a preference relation according to which people make social choices, and this preference relation should be different depending on how kind or unkind another person is perceived to have been. We test this in an experiment which is similar in spirit to [Andreoni and Miller \(2002\)](#), with subjects making choices in a modified dictator game where the price of giving varies between tasks. This allows us to test for the existence of preference relations that fit with observed choices.

Furthermore, we introduce an axiom to relate different preference relations. Reciprocity suggests behaviour changes in a particular direction depending on how kind another person has been. Our axiom gives empirical meaning to the idea that one is more or less generous depending on the behaviour of another person. The revealed preference condition we derive requires no assumptions about the functional form of the utility function or how people code behaviour as kind or unkind. We show the existence of utility functions, one of which is more selfish than the other in line with the axiom we propose, if and only if the revealed preference condition is satisfied.

The concept underlying our axiom is that two preference relations, one of which is less selfish, *agree* on particular allocations. Specifically, if some allocation x is preferred to some allocation y according to the more selfish preferences, even though allocation y

gives the decision maker more than allocation x , then x should also be preferred to y according to the more altruistic preferences. As the purely selfish motive would suggest picking y over x , choosing x means allocation y is too selfish even according to the more selfish preference relation and x should certainly be preferred to y according to the more altruistic preference relation. This *Agreement axiom* captures that while different perceptions of others' intentions may make people more or less selfish, it should not affect how they trade off fairness and efficiency.

Many behavioural findings besides reciprocity show behaviour is sensitive to changes in the choice environment that do not directly affect final outcomes, such as framing or the salience of alternatives. The Agreement axiom and its revealed preference condition can be used to model such behaviour in a very general way. Although the explanation above interprets preferences as more or less selfish in response to more or less kind behaviour, it can be applied generally whenever one preference relation likes one good more or less depending on the choice situation.

We test the revealed preference implications of our axiom in an experiment. We find that choices are indeed different depending on the behaviour of another person, that these choices can be modelled as a preference, and that they largely satisfy Agreement. This evidence suggests reciprocity is at least partially intrinsic.

2 Agreement

2.1 Preferences

Suppose a decision maker has two preference relations for allocations of money for herself and someone else. For interpretation, we assume the other to always be the same person. Let $\mathbf{X} = \mathbb{R}_+^2$ be the preference domain and for all $z = (z_1, z_2) \in \mathbf{X}$, let z_1 be the money amount she gives to herself and z_2 be the money amount for the other person. Let $\mathbf{P} = \mathbb{R}_{++}^2$ ¹ be the set of all prices. The decision maker is characterised by a pair of preference relations (\succsim_A, \succsim_S) on \mathbf{X} . The preference relation \succsim_A reflects a more altruistic

¹We take the convention that $\mathbb{R}_+^2 = \{x \in \mathbb{R}^2 : x_1 \geq 0 \text{ and } x_2 \geq 0\}$ and $\mathbb{R}_{++}^2 = \{x \in \mathbb{R}^2 : x_1 > 0 \text{ and } x_2 > 0\}$.

self and \succsim_S a more selfish self. Both preference relations are complete and transitive. ‘Altruistic’ and ‘selfish’ are taken as relative terms here; the selfish set of preferences need not be perfectly selfish.²

The decision maker faces two contexts to make choices, which we call *context A* and *context S*, where the decision maker is expected to be more altruistic in context A than in context S, for example for reasons of reciprocity. The following axiom gives the relation between these two preference relations.

Axiom 1 (Agreement). *For all $x, y \in \mathbf{X}$ with $x = (x_1, x_2)$ and $y = (y_1, y_2)$,*

$$[x \succsim_S (\succ_S) y \text{ and } x_1 \leq y_1] \text{ implies } x \succsim_A (\succ_A) y.$$

Intuitively, Agreement states that if the decision maker (strictly) prefers x to y when choosing according to \succsim_S , and she keeps less for herself in choice x than y , then she also (strictly) prefers x to y according to the more altruistic preferences (\succsim_A).³ An equivalent formulation is presented in Proposition 1 (all proofs are in the Appendix).

Proposition 1. *The Agreement axiom is equivalent to the condition that for all $x, y \in \mathbf{X}$ with $x = (x_1, x_2)$ and $y = (y_1, y_2)$, $[x \succsim_A (\succ_A) y \text{ and } x_1 \geq y_1]$ implies $x \succsim_S (\succ_S) y$.*

2.2 Empirical Implications of Agreement for Revealed Preferences

We start this section with a review of basic concepts of revealed preferences.

Definition 1. *A set of observations Ω is a finite collection of pairs $\{(z^i, r^i)\}_{i=1}^k \subset \mathbf{X} \times \mathbf{P}$.*

An observation $(z^i, r^i) = ((z_1^i, z_2^i), (r_1^i, r_2^i))$ denotes how much the decision maker chooses to keep, z_1^i , and to give to the other, z_2^i , given the choice set $\{z^i \in \mathbb{R}_+^2 :$

²Of course, a decision maker may have more than two preference relations. As the interest of this paper is in comparing preferences, we limit the analysis to two preference relations, but multiple such comparisons can be made between any number of preference relations.

³Agreement is in the same spirit as the MAT relation in Cox et al. (2008). If one preference relation is *MAT* (more altruistic than) the other preference relation according to their definition then they satisfy Agreement. For a proof, see Proposition 3 in the Appendix.

$r_1^i z_1^i + r_2^i z_2^i \leq 1\}$. We use superscripts to indicate observations and subscripts to indicate coordinates.

We now define the revealed preference relations on Ω . We say that z^i is *directly revealed preferred* to z , written as $z^i \mathbf{R}^0 z$, if $p^i z^i \geq p^i z$; it is *indirectly revealed preferred* to z , written as $z^i \mathbf{R} z$, if there exist $z^{i_1}, z^{i_2}, \dots, z^{i_m} \in \Omega$ such that $z^i \mathbf{R}^0 z^{i_1} \mathbf{R}^0 z^{i_2} \mathbf{R}^0 \dots \mathbf{R}^0 z^{i_m} \mathbf{R}^0 z$. We use \mathbf{P}^0 (\mathbf{P}) to denote the strict preference relation: z^i is *strictly directly revealed preferred* to z , written $z^i \mathbf{P}^0 z$, if $p^i z^i > p^i z$. We say z^i is *strictly revealed preferred* to z , written $z^i \mathbf{P} z$, if there exist $z^{i_1}, z^{i_2}, \dots, z^{i_m} \in \Omega$ such that $z^i \mathbf{R}^0 z^{i_1} \mathbf{R}^0 z^{i_2} \mathbf{R}^0 \dots \mathbf{R}^0 z^{i_m} \mathbf{R}^0 z$ and at least one of these revealed preference relations is strict.

Definition 2. A utility function $u : \mathbf{X} \rightarrow \mathbb{R}$ rationalises a set of observations Ω if $u(z^i) \geq u(z^j)$ whenever $z^i \mathbf{R} z^j$.

Afriat (1967) and Varian (1982) provide an easily testable condition which is necessary and sufficient for the existence of a utility function that rationalises a set of observations.

Axiom 2 (GARP). A set of observations Ω satisfies the Generalised Axiom of Revealed Preference (GARP) if for all $z^i \mathbf{R} z^j$ not $z^j \mathbf{P}^0 z^i$.

Varian (1982) provides a construction of a utility function that rationalises Ω when Ω satisfies GARP. We use the following representation that can be derived directly from Varian (1982) (a proof is in the Appendix).

Proposition 2. A set of observations Ω satisfies GARP if and only if there exists a continuous, strictly increasing and quasiconcave utility function u that rationalises Ω where $u((a, a)) = a$ for all $a \in \mathbb{R}$.

In our model, we have two sets of observations, one from context A and the other from context S, denoted by $\Omega_A = \{x^i, p^i\}_{i=1}^n$ and $\Omega_S = \{y^j, q^j\}_{j=1}^m$. Let \mathbf{R}_A and \mathbf{R}_S be the revealed preference relation on Ω_A and Ω_S respectively. If Agreement holds then the observations Ω_A (Ω_S) reveal information about \succsim_S (\succsim_A). We explain this point with Figure 1.

In the left image, given the budget line in context A, the decision maker chooses x^i , so x^i is revealed preferred to all allocations on and below the budget line in context A. Every allocation from the area indicated by the red vertical lines gives the decision maker less

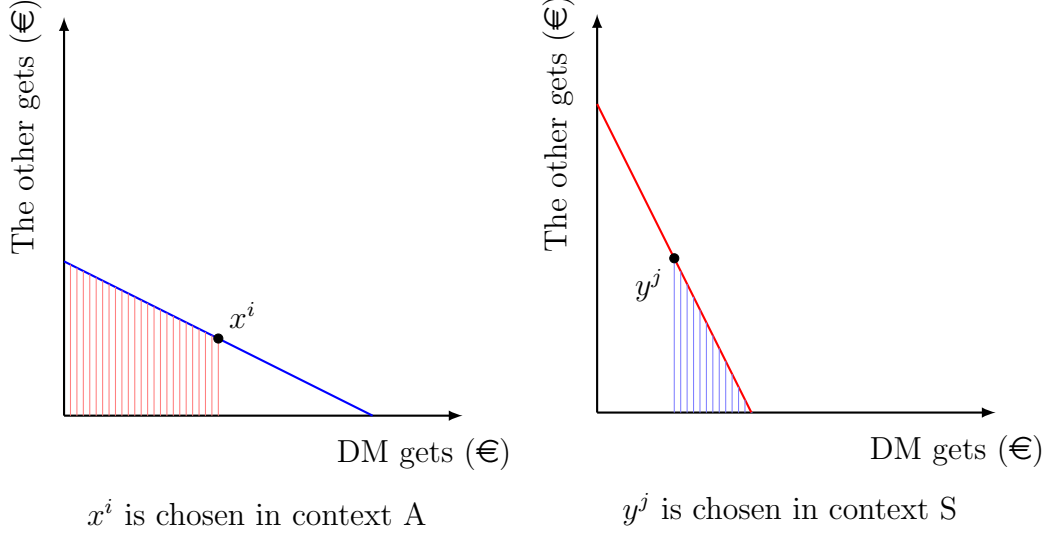


Figure 1: Revealed preference implications of the Agreement axiom. The observations are revealed preferred to the area below the budget line in the same context and to the area indicated with the vertical lines in the other context.

than x^i does. Thus, by Agreement, x^i is also revealed preferred to the allocations in the area indicated by the red vertical lines in context S. In the right graph, the decision maker chooses y^j in context S, and any allocation in the area indicated with the blue vertical lines gives the decision maker more than y^j does. Thus, by Agreement, she should also prefer y^j over any allocation in the blue area in context A.

Through Agreement we can thus extend the revealed relation \mathbf{R}_A by incorporating information from Ω_S and extend \mathbf{R}_S by incorporating information from Ω_A . Let $\Omega = \Omega_A \cup \Omega_S = \{z^i, r^i\}_{i=1}^k = \{x_i, p^i\}_{i=1}^n \cup \{y_j, q^j\}_{j=1}^m$, where (z^i, r^i) is an observation from either Ω_A or Ω_S . Writing these extensions as $\tilde{\mathbf{R}}_A$ and $\tilde{\mathbf{R}}_S$, it follows that $z^i \tilde{\mathbf{R}}_A^0 z^j$, if $z^i \mathbf{R}_A^0 z^j$ or if $z^i \mathbf{R}_S^0 z^j$ and $z_1^i \leq z_1^j$; and that $z^i \tilde{\mathbf{R}}_S^0 z^j$, if $z^i \mathbf{R}_S^0 z^j$ or if $z^i \mathbf{R}_A^0 z^j$ and $z_1^i \geq z_1^j$. $\tilde{\mathbf{P}}_A$ and $\tilde{\mathbf{P}}_S$ are defined analogously. We next define rationalisation in our model.

Definition 3. An altruistic utility function $u : \mathbf{X} \rightarrow \mathbb{R}$ and a selfish utility function $v : \mathbf{X} \rightarrow \mathbb{R}$ Agreement-rationalise (AG-rationalise) Ω , if $u(z^i) \geq u(z^j)$ whenever $z^i \tilde{\mathbf{R}}_A z^j$ and if $v(z^i) \geq v(z^j)$ whenever $z^i \tilde{\mathbf{R}}_S z^j$.

The altruistic and selfish utility functions represent the extended revealed preference relations inferred from the observations Ω . AG-rationalisation captures what it means to

choose according to one's preferences if these preferences satisfy Agreement.

Axiom 3 (AG-GARP). *A set of observations Ω satisfies Agreement-GARP (AG-GARP), if $z^i \tilde{\mathbf{R}}_A z^j$ implies not $z^j \tilde{\mathbf{P}}_A^0 z^i$ and $z^i \tilde{\mathbf{R}}_S z^j$ implies not $z^j \tilde{\mathbf{P}}_S^0 z^i$.*

Theorem 1. *Given a set of observations $\Omega = \Omega_A \cup \Omega_S$ with $\Omega_A = \{x^i, p^i\}_{i=1}^n$ and $\Omega_S = \{y^j, q^j\}_{j=1}^m$, the following conditions are equivalent:*

- (a) *The set of observations Ω satisfies AG-GARP.*
- (b) *There exist an altruistic utility function u and selfish utility function v that are continuous, strictly increasing and quasiconcave and AG-rationalise Ω . Moreover, for all $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq v(y)$ and for all $x, y \in \mathbf{X}$, $v(x) \geq v(y)$ with $x_1 \leq y_1$ implies that $u(x) \geq u(y)$.*

Theorem 1 describes that if the set of observations satisfies AG-GARP, the decision maker's choices can be represented by two utility functions, one of which is more altruistic than the other. These two utility functions represent the extended revealed preference relations. Theorem 1 thus captures the empirical implications of Agreement and rationalisation.

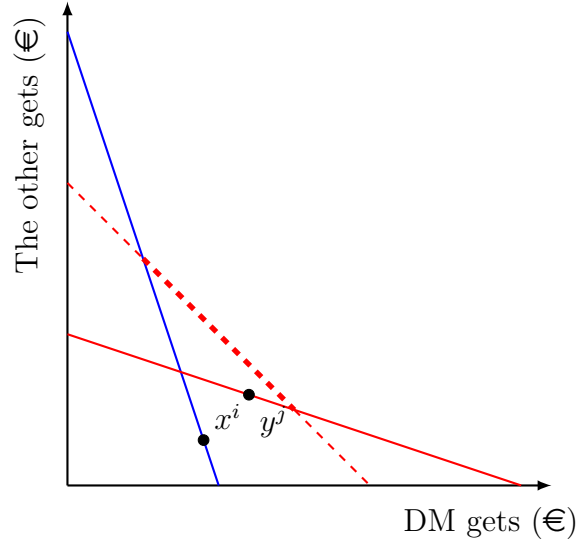


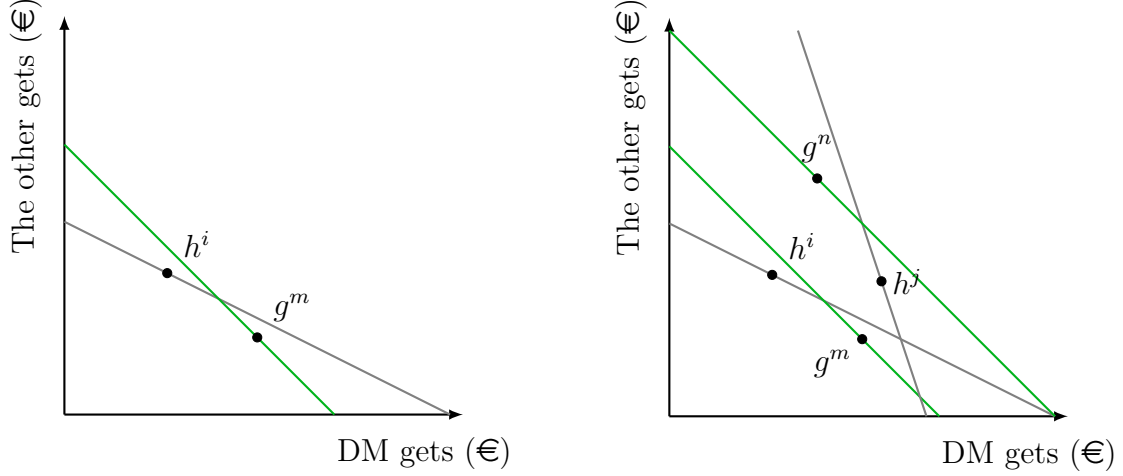
Figure 2: AG-GARP predicts that a third choice on the dashed red budget line must be on the thick red part.

We can use AG-GARP to predict choices. This is illustrated in Figure 2. x^i is chosen in context A and y^j is chosen in context S. Suppose a person is next presented with the dashed red budget in context S. A person who chooses according to preferences which satisfy AG-GARP must then choose on the thick red part of the budget line. Choosing to keep more violates GARP within context S (and therefore violates AG-GARP); choosing to keep less violates Agreement.⁴ Thus, we get predictions beyond that choices must be different or that the decision maker must keep more in the selfish context on the same budget.

We get clear welfare implications from AG-GARP using the behavioural welfare definitions of Bernheim and Rangel (2007). They define an alternative x to be a *strict individual welfare improvement* over an alternative y if a decision maker sometimes chooses x but never y , or never chooses either, when x and y are both available. When $z^i \mathbf{P}_S z$ and $z_1^i \leq z_1$, we have $z^i \tilde{\mathbf{P}}_A z$ and $z^i \tilde{\mathbf{P}}_S z$. A decision maker who satisfies AG-GARP will then never choose z over z^i , hence z^i is a strict individual welfare improvement over z . Similarly, if $z^i \mathbf{P}_A z$ and $z_1^i \geq z_1$, we have $z^i \tilde{\mathbf{P}}_A z$ and $z^i \tilde{\mathbf{P}}_S z$ and z^i is a strict individual welfare improvement over z for a decision maker who satisfies AG-GARP.

Although we have assumed that we have a dataset from context A where the decision maker is more altruistic and a dataset from context S where the decision maker is more selfish, it is not essential that we know for sure in which context the decision maker is more altruistic. If we want to model changing preferences without specifying in which context the preferences should be more or less altruistic we can let the data speak for itself (if we have two distinct sets of data). Satisfying Agreement means violations of GARP can only go in one direction (see Figure 8 in the Appendix). We can use this to infer from the data which is context A and which is context S. If two observations from two different contexts violate GARP, then the observation where the decision maker keeps less must come from context A, and hence all observations from that set must belong to Ω_A , as illustrated by the left image of Figure 3. If we do not wish to assume which set of observations is more altruistic, we can also still test for AG-GARP: AG-GARP is then violated if we find violations of GARP that go in different directions. This is illustrated

⁴Such predictions can easily be extended to situations where a person does not choose in perfect accordance with AG-GARP by correcting for efficiency. Efficiency is discussed in Section 4.2.



h^i must be context A, g^m must be context S Cannot be rationalised with Agreement

Figure 3: Identifying contexts (preference relations) and testing AG-GARP without assuming which context is A or S.

by the right image of Figure 3 for two sets of observations, $\{h^i, h^j\}$ and $\{g^n, g^m\}$. The violation of GARP by h^i and g^m suggests the set $\{g^n, g^m\}$ must come from the more selfish context, but the violation of GARP by h^j and g^n suggests $\{h^n, h^m\}$ must be the observations from the more selfish context. This violates AG-GARP no matter which set of observations is taken as the more selfish one.

In our setting, a decision maker has preferences over $X = \mathbb{R}_+^2$ and every point in X is taken as an allocation of money. The preference domain X can have other interpretations. For example, $x = (x_1, x_2) \in X$ can be consumption and leisure, and a person may prefer leisure (good 1) more when they are older than younger, or more in the summer than in the winter. Our method can be applied very generally.

3 Experiment

3.1 Experiment design

To measure how social preferences depend on the perceived kindness of another person we ran an interactive experiment where pairs of subjects made choices. One player made a single Choice (hence we call this player ‘player C’ or simply ‘C’) between two allocations:

either giving €13 to the other player and keeping €27 or giving €18 and keeping €18. In our experiment, these two possibilities represent the contexts A and S that the second player faced. Using the strategy method, the second player made 14 choices from 14 linear Budgets (hence, we call this player ‘player B’) for each of these two contexts (thus making a total of 28 choices). For the purposes of this paper, we are only interested in the choices made by player B. Player C was only part of the experiment to incentivise player B.

Crucially, both players were informed that only one choice would be implemented for real: either the choice of the first player (C) would be implemented or one of the choices of player B would be implemented. In the latter case, one budget would be randomly selected from the set of budgets corresponding to the choice made by C. Thus, Player B’s choice was only practically relevant as a response to C’s intention, never to a practically implemented choice by C.

Player C was informed that player B could divide money between them at different rates, that the minimum B could give them was €0 and that the maximum differed per budget but was never more than €60. Player C was informed that B made 14 choices for both of C’s possible choices. Player B was informed that player C was presented with this information. Both players were informed how they would be matched to each other and were informed about the payment procedure, including that both players would be paid either according to C’s choice or to one of B’s choices and that this was determined randomly.

After the instructions, both player B and player C were asked to answer three multiple choice comprehension questions. If they answered a question incorrectly they were given immediate feedback as to why their answer was wrong. They could only continue once they had answered the question correctly. Player B was also given some practice tasks to familiarise them with the interface.⁵

Player C could indicate their choice simply by selecting the desired allocation and confirming their choice (they could revise their choice before confirming). The order of the alternatives was randomised and the same in the instructions and in the actual choice

⁵Player C was not provided with practice tasks because their task was a very simple binary choice which required no more than clicking on the desired option.

situation.

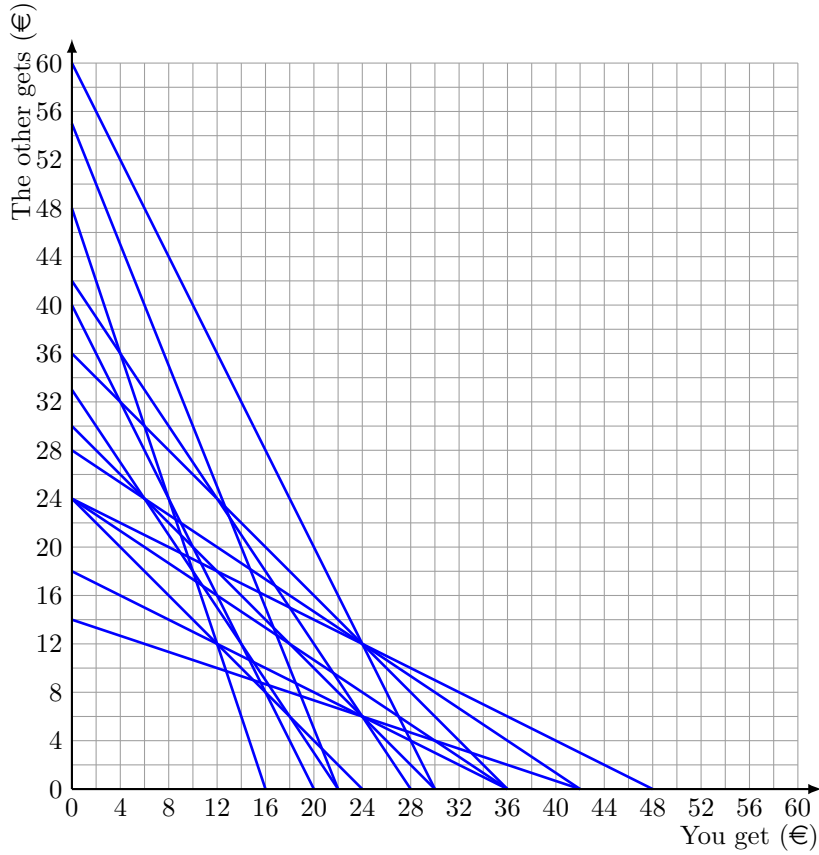


Figure 4: Budgets used in the experiment

The 14 budgets player B was faced with in each context (shown in Figure 4) were all linear budgets where B could give some money to C at different rates, from giving C €0.33 to giving C €3 for every euro B gave up. The minimum B could keep was always €0, as was the minimum B could give away to C. The maximum amount B could keep or give to C varied by budget and was never higher than €48, respectively €60. Budgets were chosen such that they intersected at many points to get good test power. The average slope was bigger than 1, meaning that on average giving up €1 increased C's payoff by more than €1, to make it attractive for player B to give at least some money to C (if all choices are on the axis, test power is zero).

Which of the contexts (player C's two possible choices) for which player B made choices from budgets was presented first was randomised at the start of the experiment and then kept the same in the instructions and in the choice tasks. The order of the

14 budgets was randomised for each of the contexts separately, so that the order of the budgets was different between the two contexts.

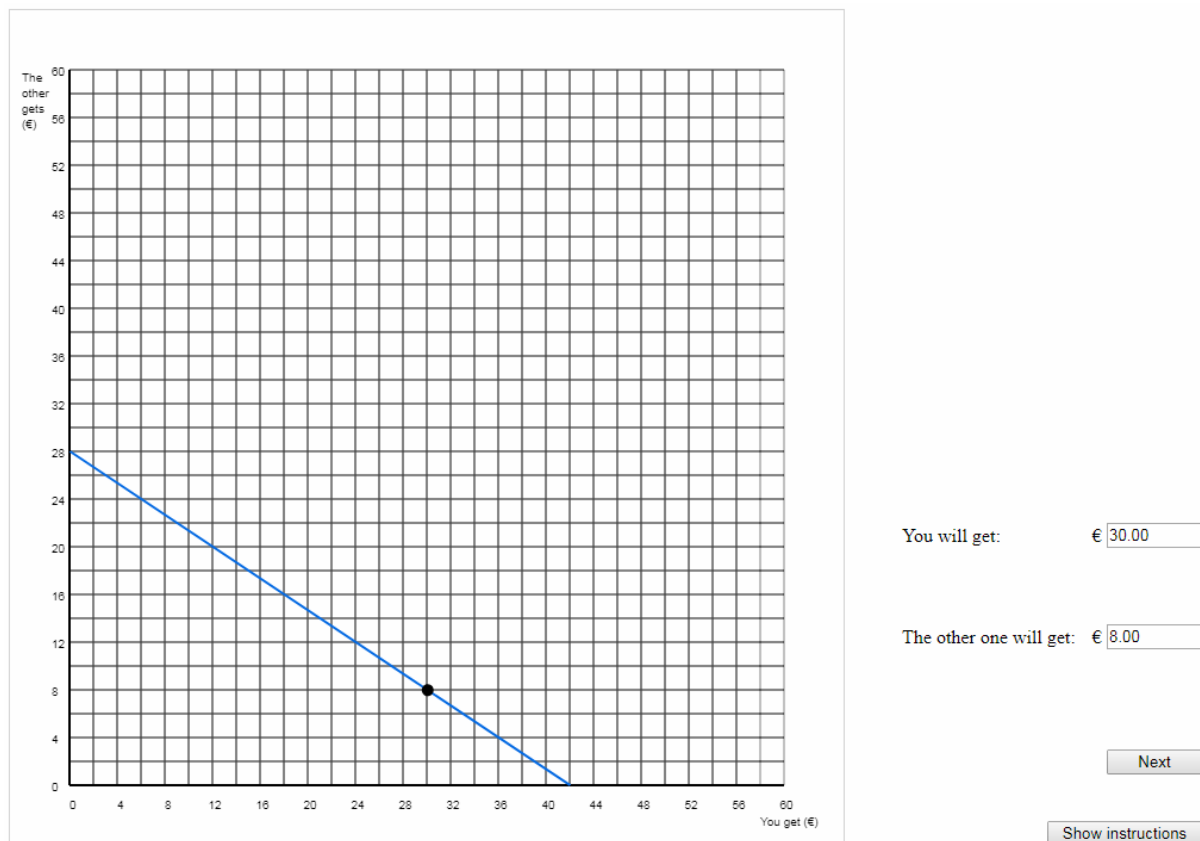


Figure 5: A screenshot of the interface for one of the player B’s tasks.

To make performing the tasks as easy as possible for player B we developed an interface which graphically displayed the budget (shown in Figure 5). Player B could make choices by clicking on any point on the budget, by typing the amount they wanted to keep, or by typing the amount they wanted to give to player C. When either of these three methods had been used, a dot would appear on the budget line to indicate their choice and the amounts that player C and B would receive were displayed automatically in number fields. Player B could then revise their choice if they were not happy with the resulting allocation by clicking somewhere else on the budget line, by dragging the dot around on the budget line, or by entering in a different number in either of the fields displaying how much they would keep or give away.

The software automatically calculated a minimum step size in multiples of €0.05 and any choice was automatically converted to the closest step. This ensured that all choices

were exactly on the budget line. For example, where player B could give away €3 for every €1 they gave up, the minimum amount B could give up other than 0 was €0.05 and the minimum amount they could give away other than 0 was €0.15. Giving up €0.03 or giving away €0.10 to C was automatically rounded to giving up €0.05 and giving away €0.15. Entering an amount not on the budget line (negative amounts or amounts greater than the maximum amount that could be kept or given to C) resulted in a message indicating that the amount entered was invalid. Player B could only continue to the next task after choosing a valid allocation.

On entering the lab, subjects were assigned a cubicle and asked to wait for the start of the experiment (if there was an odd number of subjects the last subject to arrive was given a show-up fee and did not participate). At the start of the experiment, subjects were handed an envelope containing two slips of paper containing their subject ID. After using their subject ID to log on, we collected their envelopes, leaving one of their IDs at their cubicle, and sorted them into one pile containing player C IDs and one containing player B IDs. Subjects who finished early were then asked to randomly match players by choosing an envelope from each pile (without seeing the ID codes within) and to put the ID code of player B into the envelope containing the ID of player C. Next, subjects were asked to roll a die to determine whether each pair would be paid according to player C or player B's choice (where the probability of C or B being selected was equal). This was marked on the envelope. If the pair was paid according to B's choice, a subject was asked to draw a ball from a bag with 14 balls numbered 1 to 14 inclusive to select according to which budget the pair would be paid out. This too was marked on the envelope. When all subjects had finished subjects were asked one by one to come to the front desk to be paid.

The experiment was run in the ESE-econlab of Erasmus University Rotterdam. There were 9 sessions of roughly 20 subjects each, with a total of 170 subjects participating. The experiment lasted about an hour and the average payment was €16.47.

4 Analysis

In this section, we first show a few descriptive statistics for the different treatments. We then present our analysis of the data based on the revealed preference approach outlined in Section 2, and finally we present the results of more conventional parametric analysis assuming a CES utility function.

4.1 Descriptives

Player C chose the more selfish allocation (€27 for themselves, €13 for player B) roughly as often as the more equal allocation (€18, €18), with frequencies of 55% and 45%, respectively. This shows both options were attractive to player C, which is important because if one option was very unattractive and therefore unlikely to be chosen, this case would not be well-incentivised for player B.

Table 1: Choices by players B conditional on the choice by C.

	C kind			C unkind		
	Self (€)	Other (€)	Share self (%)	Self (€)	Other (€)	Share self (%)
Min	5	0	18.9	6	0	25.0
Median	19	10	70.4	22	6.90	78.3
Max	48	40	100	48	32.25	100

Throughout the analysis, we will treat player C choosing €27 for themselves and €13 for player B as player C being (relatively) unkind, and player C choosing €18 for themselves and player B as C being (relatively) kind. Table 1 shows various statistics on the choices of players B conditional on the choice by player C. The median choice gave away about 30% of their endowment to player C if C is kind, and about 22% if C is unkind. The median amount kept by player B was about €3 higher and B gave C about €3 less if C is unkind. These differences are all significant with p -values < 0.001 according to Wilcoxon signed-rank tests.⁶ Player B is thus less generous when C chooses allocation

⁶This is true both when we take each individual choice as an observation, and when we take the

(€27, €13) than when C chooses (€18, €18), consistent with reciprocity if we code the former as being a less kind action. For the budgets with a price of 1, corresponding to the classic dictator game, the share of what was kept is 67.8% when C is kind and 77.3% when C is unkind.

4.2 Revealed preference analysis

In our analysis, we only use the data of player B's choices. We first present the revealed preference analysis. We take the set of player B's choices responding to player C's possible selfish choice of keeping €27 and giving €13 as context S and the set of player B's choices responding to player C's possible more altruistic choice of keeping €18 and giving €18 as context A. Based on Proposition 2 and Theorem 1, we have the following hypotheses.

Hypothesis 1. *Player B has different preferences in both contexts: $\succsim_A \neq \succsim_S$. This implies the following testable conditions: GARP holds on Ω_A and on Ω_S , but not on Ω .*

Hypothesis 1 is the simplest hypothesis generated by reciprocal preferences: people choose according to some (social) preference relation, but this preference relation is different depending on how kind another player has been. This requires GARP to be satisfied on the data from each separately, as otherwise people are not maximising any utility function, but not on the data together, meaning behaviour is not consistent with people maximising the same utility function in both contexts.

Hypothesis 2. *Player B has different preferences in each context, which are connected by Agreement. Player B uses preferences \succsim_A in context A and \succsim_S in context S. Testable condition: AG-GARP holds on Ω , with the choices from Ω_S being more selfish.*

This second hypothesis captures that reciprocal preferences have a direction: we expect a person to be more generous when the recipient has been more kind. The Agreement axiom as captured by AG-GARP gives empirical meaning to being more generous (or more selfish) for budgets with different prices and endowments (see Section 2).

As satisfying GARP is a binary variable, where one either satisfies it or does not and minor and economically unimportant violations of GARP are treated the same as very

average per subject as an observation.

significant violations, we use Afriat’s *Critical Cost Efficiency Index* (CCEI) (Afriat, 1972) instead. This captures how far away someone’s choices are from utility maximisation. The index is bounded between 0 and 1. When choices satisfy GARP, the index is 1. When GARP is violated, the index is smaller than 1, with bigger violations (relative to the budget) resulting in lower indices. We construct a similar index for AG-GARP, which we call the *Agreement Efficiency Index* (AGEI) or simply Agreement efficiency.

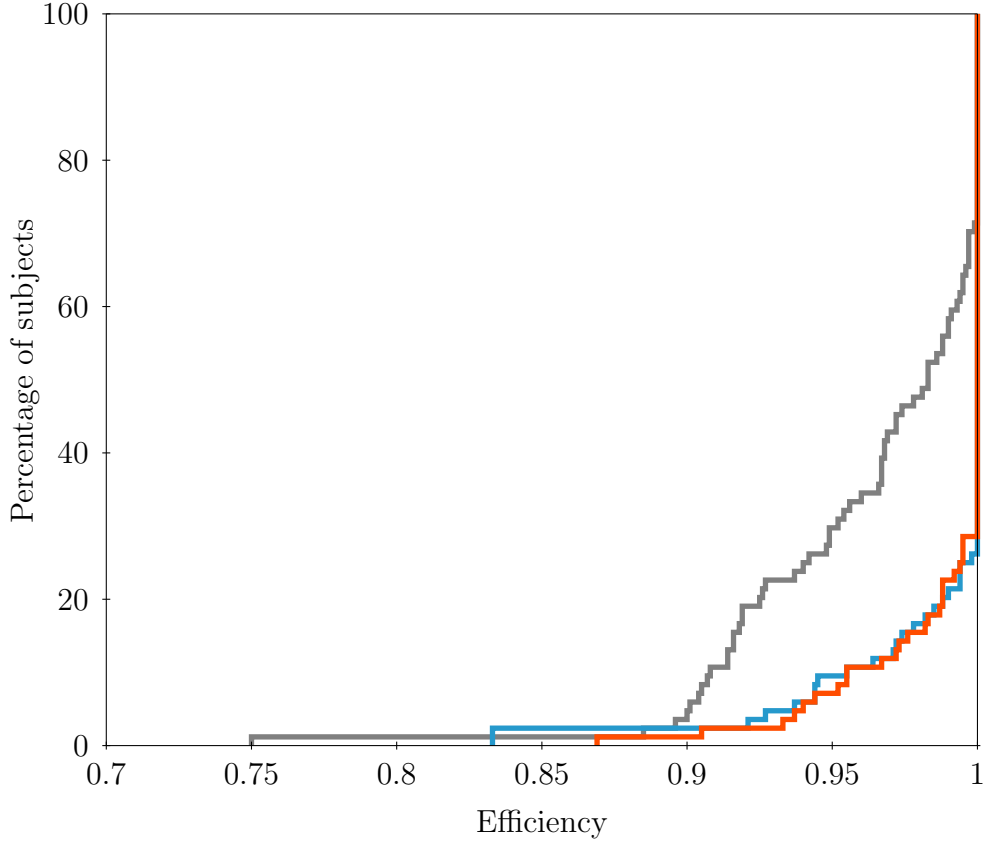


Figure 6: Empirical CDFs of CCEI (efficiency) for context A (blue), S (red) and a mixture of choices from A and S (grey).

Figure 6 shows empirical cumulative distribution functions of CCEIs for choices made in context A (in blue) and S (red). The distributions are very similar, which means subjects came equally close to utility maximisation in either treatment. A Wilcoxon signed-rank test (used throughout this section) cannot reject that the distributions are the same (p -value 0.703). The grey line in Figure 6 shows CCEIs when combining choices from half the budgets in context A with choices the same subject made from the remaining

budgets in context S.⁷ These efficiencies are calculated based on the same budgets and the same number of choices, hence power is the same. The lower efficiencies observed here (p -value < 0.001 compared to either A or S) are evidence that preferences revealed in context A and S are different. Because preferences differ between the two contexts, revealed preferences are contradictory, which leads to violations of GARP and low CCEIs when we treat them as if they come from the maximisation of a single utility function.

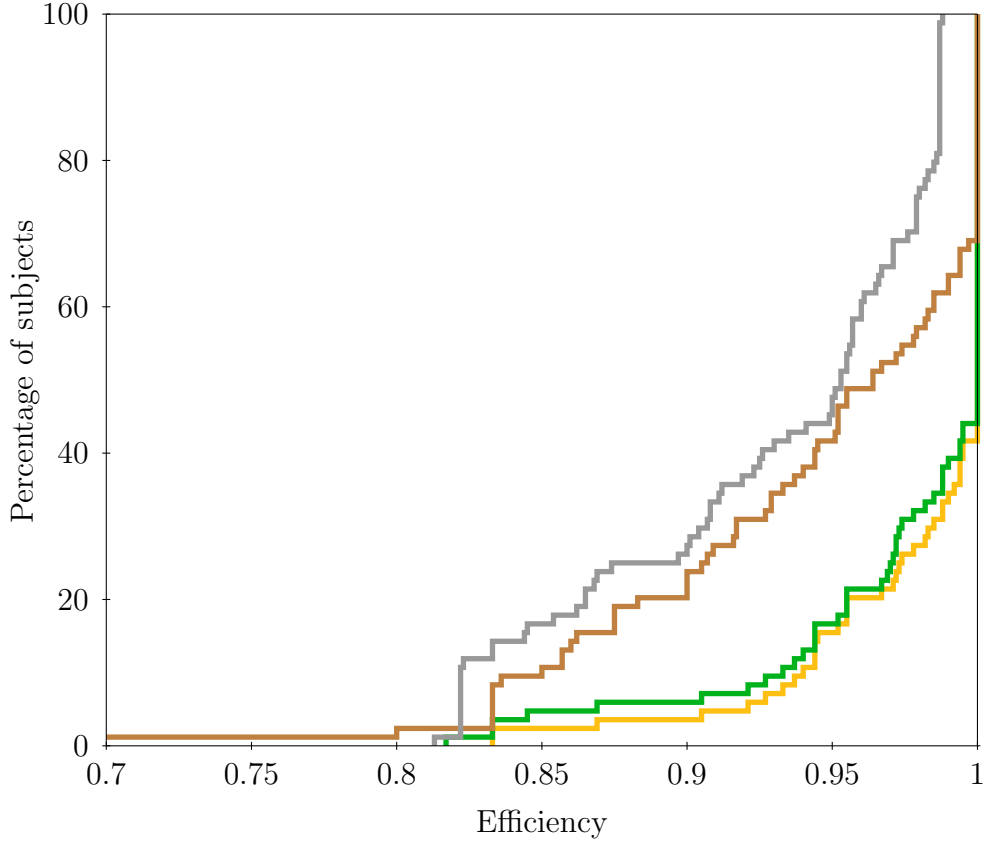


Figure 7: Empirical CDFs of the minimum CCEI from context A and S (yellow), Agreement efficiency over set A and S (green), Agreement efficiency with set A and S reversed (brown) and Agreement efficiency over observations from A of two different subjects (grey).

Now that it has been established that revealed preferences are different between context A and S we investigate whether these different revealed preferences can be connected

⁷There are many different ways to select these budgets. The grey line in Figure 6 is the mean of CCEIs calculated across these different combinations.

with Agreement. In Figure 7 the empirical cumulative distribution function of Agreement efficiencies (taking the preference relation from the kind treatment as the more altruistic one) is presented in green, together with the empirical distribution function of the minimum Afriat efficiency of context A and S (yellow). The latter is an upper bound on the Agreement efficiency: any violation of GARP in either A or S is also a violation of AG-GARP.⁸ As we can see, the distributions of Agreement efficiencies and Afriat efficiencies are very close, so Agreement fits behaviour very well. The brown line shows Agreement efficiencies when we take the data from the unkind treatment as the more altruistic data and the data from the kind treatment as the more selfish data. Agreement efficiencies are then clearly lower (p -value < 0.001) than the Agreement efficiencies of the green line, which is evidence that indeed people become more generous in response to a kind action by another person.

One possible explanation for the small difference between Agreement and Afriat efficiencies is that we may have little power to reject AG-GARP. Revealed preference conditions are very general and therefore tend to be rather permissive, meaning that we may not detect violations of them even if the decision maker does not satisfy the conditions. The decrease in Agreement efficiencies when we reverse the altruistic and selfish data shows that the condition is not so weak that is unlikely to detect violations. This is a somewhat extreme case, taking data where average giving is lower as the more altruistic data. Therefore Figure 7 also presents (in grey) Agreement efficiencies calculated taking the choices of one subject from context A and the choices of another subject from the same context.⁹

We do not expect the altruistic choices of one subject to be a more selfish version of the choices by another subject, so we expect to detect violations of AG-GARP if AG-GARP is sufficiently demanding. The Agreement efficiencies of the grey line in Figure 7 are clearly much worse (p -value < 0.001), so the good performance of Agreement is

⁸For this reason, the difference is almost necessarily statistically significant (p -value < 0.001).

⁹Because the issue here is that two different conditions are tested (GARP and AG-GARP) rather than that the number of budgets differs (as in Figure 6) we cannot simply correct for the number of budgets. There are many ways to match one subject to another subject; we report the mean taken over the Agreement efficiencies calculated for every possible match.

not due to a lack of power, but simply because it describes behaviour well. There is a direction to how preferences change depending on the kindness of the other person, and this can be captured with Agreement.

4.3 Parametric analysis

The Agreement axiom can also be used with parametric assumptions. In this section, we perform parametric analysis to complement the revealed preference analysis of Section 4.2. We do so for constant elasticity of substitution (CES) utility functions. The Agreement axiom has an intuitive interpretation for CES utility functions, which have the following form:

$$u(z) = (\alpha z_1^\rho + (1 - \alpha) z_2^\rho)^{1/\rho} \quad (1)$$

Here $z \in \mathbf{X}$ is an allocation. Parameter ρ determines the elasticity of substitution, that is, the curvature of the indifference curves. Thus ρ determines the trade-off between equality and efficiency. Parameter α is the distribution parameter, and captures how the payoff to the decision maker is traded off against the payoff of the other person. If decision makers' utility can be described by (1), that is, $u(z) = (\alpha_A z_1^{\rho_A} + (1 - \alpha_A) z_2^{\rho_A})^{1/\rho_A}$ for \succsim_A and $v(z) = (\alpha_S z_1^{\rho_S} + (1 - \alpha_S) z_2^{\rho_S})^{1/\rho_S}$ for \succsim_S , the hypotheses parallel to those of our revealed preference analysis are the following (the proof is in the Appendix).

Hypothesis 3. *Player B has different preferences in each context, which are connected by Agreement. Testable parametric implication:*

$$\alpha_A \leq \alpha_S \quad \text{and} \quad \rho_A = \rho_S.$$

Following Andreoni and Miller (2002), we estimate CES functions for each individual for both treatments. With the budget constraint $z_1 + p z_2 = m$ the demand function is

$$z_1(p, m) = \frac{[\alpha/(1 - \alpha)]^{1/(1-\rho)}}{p^{-\rho/(1-\rho)} + [\alpha/(1 - \alpha)]^{1/(1-\rho)}} m = \frac{D}{p^r + D} \quad (2)$$

Where $r = -\rho/(1 - \rho)$ and $D = [\alpha/(1 - \alpha)]^{1/(1-\rho)}$. We first estimate r and D by non-linear least squares, then back out estimations of α and ρ .

For only 59 out of the 85 player B we can fit the CES utility function. Twelve subjects make all choices on one axis (in all cases keeping everything) in either treatment and are

therefore excluded, leaving 73 subjects. Additionally, the estimation for 14 player Bs does not converge because they chose on the axis except once or twice in each treatment, or they always chose proportionally in one treatment (the ratio of the money amount player B keeps relative to the amount they give is constant).

The median $\hat{\alpha}$ and $\hat{\rho}$ for both contexts are reported in Table 2: $\hat{\alpha}$ is bigger when player C chooses the less generous allocation than when C chooses the more generous allocation. A Wilcoxon signed-rank test indicates that this difference is significant (p -value 0.033). By contrast, the difference in $\hat{\rho}$ between the two contexts is not significant (p -value 0.188). This is in accordance with hypothesis 3.

Table 2: Summary of Wilcoxon Signrank Test

	Context A	Context S	p -value
Median $\hat{\alpha}$	0.85	0.98	0.033
Median $\hat{\rho}$	-0.91	-1.00	0.188

5 Discussion

For budgets with a price of 1, corresponding to the classic dictator game, we find that our player B keeps 67.8% when C is kind and 77.3% when C is unkind, meaning they give away 27.4% of their endowment on average. This is close to giving in the typical dictator game at 28.3% (Engel, 2011). The share that is given away in either of our treatments separately (32.2% for the kind and 22.7% for the unkind treatment) is also well within the range of typical observations (cf. Engel, 2011, p. 589).

Like Andreoni and Miller (2002) we find that subjects largely satisfy utility maximisation or come very close to maximising utility when making social choices on linear budgets. We find more violations of GARP, which can be explained by the greater number of budgets we use (14 in either context rather than 8). The Afriat efficiencies we find are slightly higher than those found in the social choice experiment with linear budgets by Fisman et al. (2007), who find that only 54% of subject achieve an efficiency of 1 compared to around 74% of our subjects, which again is probably due to a difference in

the number of budgets (50 in the case of [Fisman et al., 2007](#)).

Many studies have reported findings of reciprocal behaviour (as briefly discussed in the Introduction). Some studies (e.g. [Reuben and Suetens, 2012](#); [Cabral et al., 2014](#)) have also looked into the frequency of behaviour consistent with a strategic motive relative to the frequency of behaviour without a strategic motive. However, none of these studies test whether their findings are consistent with a preference for reciprocity, which requires that concerns for reciprocity can be expressed as a well-behaved preference ordering. Testing this hypothesis is only possible with intersecting budgets such as the ones presented in [Figure 4](#). Essentially, without such intersecting budgets, it is not possible to distinguish between intrinsic reciprocity (a preference) and *strong reciprocity*, a tendency to reward kind behaviour and punish unkind behaviour ([Fehr et al., 2002](#)). Strong reciprocity may reflect a preference (intrinsic), but it may also be the consequence of following a social norm or a heuristic (as pointed out by [Reuben and Suetens, 2018](#)), because of the salience of actions taken by others, or for some other reason.

The revealed preference condition we introduce and test is necessary *and sufficient* for the existence of different (sub)utility functions depending on the kindness of the other player, with the decision maker being more generous when the other player is more kind, and more selfish when the other player is less kind. This means that the condition *exhausts* the empirical implications of a preference for reciprocity.¹⁰

A decision maker having different preferences depending on the behaviour of others means there are multiple preference relations according to which a person may choose. In that sense, it is similar to random utility models. As our Agreement axiom essentially characterises a single-crossing condition, the work on random utility models most closely related to our paper is [Apesteguia et al. \(2017\)](#). Whereas they focus on the stochastic choices function for random utility models with single-crossing utility, our result is on the revealed preference implications and utility representation of single-crossing utility as captured by our Agreement axiom. [Adams et al. \(2017\)](#) have a similar focus on the revealed preference implications of different preference relations, but their model imposes no restrictions on the data (‘anything goes’). Because our method allows for classifying different revealed preference relations, it is also related to [Crawford and Pendakur \(2013\)](#)

¹⁰Apart from how beliefs are formed about the kindness of another player.

and [Castillo and Freer \(2018\)](#). They focus on classifying groups of different preferences relations, whereas we also have results on how these different preferences are related.

[Cox et al. \(2008\)](#) consider the nonparametric implications of different social preference relations, under the assumption of the existence of differentiable utility functions that represent preferences. If one preference relation is MAT (more altruistic than) the other preference relation according to [Cox et al.’s \(2008\)](#) definition, then the two preference relations satisfy our Agreement axiom. We do not assume the existence of utility functions (differentiable or otherwise), but instead start from a behavioural axiom and derive a revealed preference condition which is equivalent to the existence of utility functions that satisfy our axiom.

6 Conclusion

We introduce a new axiom which we call Agreement to give empirical meaning to the idea that one preference relation is more generous (or more selfish) than another preference relation. The revealed preference condition we derive from this axiom generalises GARP when preferences are context-specific, but where preferences in one context are informative about preferences in the another context. We show that if and only if data satisfies our revealed preference condition, there exist utility functions, one of which is more selfish than the other, that represent the preferences revealed by someone’s choices. The Agreement axiom and the revealed preference implications we derive can be used to model changing preferences whenever someone likes some particular good better in one context than another, not only in social choice situations. Our revealed preference method allows for predicting choices in one context based on choices observed in a different context and for drawing conclusions about welfare.

Applying our revealed preference results to a social choice experiment, where the context was generated by the kindness of another player, we find that people have consistent preferences for a given level of kindness of the other player, but that their preferences are different for different levels of kindness. Furthermore, we find that choices are largely consistent with Agreement. Our findings provide support for the interpretation of reciprocal behaviour as reflecting a preference.

7 Appendix: Proofs

Proof of Proposition 1. First, we show Agreement implies the alternate version. Proof by contradiction. Suppose there exist x', y' such that $x' \succsim_A y'$ and $x'_1 \geq y'_1$ but $y' \succ_S x'$. Then by Agreement $y' \succ_A x'$, a contradiction of the assumed preference relation $x' \succsim_A y'$. The proof that the alternative formulation implies Agreement works analogously. \square

Proof of Proposition 2. Since Ω satisfies GARP, by Varian (1982), there exist U^i and $\lambda^i > 0$, $i = 1, \dots, n$ such that for all $x \in \mathbb{R}_+^2$,

$$U(x) = \min_{i \leq n} (U^i + \lambda^i p^i(x - x^i))$$

Given $x \in \mathbb{R}_+^2$, $u : \mathbb{R}_+^2 \rightarrow \mathbb{R}$, such that $U(u(x), u(x)) = U(x)$. Then u is the certainty equivalent (CE) function that represents the same preference as U , moreover, for every $x \in \mathbb{R}_+^2$, $\min\{x_1, x_2\} \leq u(x) \leq \max\{x_1, x_2\}$. Now we show that u is continuous, strictly increasing and quasiconcave, which is equivalent to showing that \succsim represented by U is continuous, strictly increasing and convex.

(1) U is continuous, so \succsim is continuous.

(2) Let $x = (x_1, x_2) \in \mathbb{R}_+^2$ and $\epsilon > 0$, then for all i

$$U^i + \lambda^i p^i((x_1 + \epsilon, x_2) - x^i) = U^i + \lambda^i p^i(x - x^i) + \lambda p^i(\epsilon, 0) > U^i + \lambda p^i(x - x^i)$$

Then,

$$\min_{i \leq n} (U^i + \lambda p^i((x_1 + \epsilon, x_2) - x^i)) > \min_{i \leq n} (U^i + \lambda p^i(x - x^i))$$

That is, $x + (\epsilon, 0) \succ x$. Similarly, we have $x + (0, \epsilon) \succ x$ for all $\epsilon > 0$.

(3) Given $x, y \in \mathbb{R}_+^2$ with $U(x) = U(y)$ and $\alpha \in (0, 1)$, then

$$\begin{aligned} U(\alpha x + (1 - \alpha)y) &= \min_{i \leq n} (U^i + \lambda p^i((\alpha x + (1 - \alpha)y) - x^i)) \\ &\geq \alpha \min_{i \leq n} (U^i + \lambda p^i(x - x^i)) + (1 - \alpha) \min_{i \leq n} (U^i + \lambda p^i(y - x^i)) \\ &= U(x) = U(y) \end{aligned}$$

Thus, \succsim is convex.

Therefore, u is continuous, strictly increasing and quasiconcave. □

In our model, Ω is two-dimensional. Then GARP can be simplified by the following Theorem.

Theorem 2 (Theorem 1 of Banerjee and Murphy 2006). *Given $\mathbf{X} \subset \mathbb{R}_+^2$, GARP is equivalent to that $x^i \mathbf{R}^0 x^j$ not $x^j \mathbf{P}^0 x^i$.*

Let $N_\varepsilon(x) = \{a \in \mathbf{X} : d(a, x) < \varepsilon\}$ be the open neighbourhood of x , and let d be the Euclidean distance. The interior of a set S , $\text{int } S$, is the set of all $a \in S$, such that there exists ε_a such that $N_{\varepsilon_a}(a) \subset S$. The boundary of a set S is $\text{d}S = \{a \in S : a \notin \text{int } S\}$. For a finite set $\{s^1, \dots, s^n\} = S \in \mathbf{X}$, the *convex hull* of S is

$$CH(S) = \{a \in \mathbf{X} : a = \sum_{i=1}^n \lambda^i s^i, \sum_{i=1}^n \lambda^i = 1, \lambda^i \in [0, 1] \text{ for all } i\}.$$

The *convex monotonic hull* of S is,

$$CMH(S) = CH(\{a \in \mathbf{X} : a \geq s^i \text{ for some } i = 1, \dots, n\}).$$

Let $Z = \{z^i\}_{i=1}^k = \{x_i\}_{i=1}^n \cup \{y_j\}_{j=1}^m$, and $P = \{r^i\}_{i=1}^k$ be the set of corresponding prices to Z and $\mathbf{B}(r^i) = \{a \in \mathbf{X} : a \cdot r^i < 1\}$. We define $RW(\tilde{\mathbf{R}}_S^0, z^m)$ to be the revealed worse set of observation (z^m, r^m) according to the revealed preference \tilde{R}_S . Specifically, if $(z^m, r^m) \in \Omega_S$, $RW(\tilde{\mathbf{R}}_S^0, z^m) = \{a \in \mathbf{X} : a \cdot r^i < 1\}$ and if $(z^m, r^m) \in \Omega_A$, $RW(\tilde{\mathbf{R}}_S^0, z^m) = \{a \in \mathbf{X} : a \cdot r^i < 1, a_1 \leq z_1^m\}$. Before proving Theorem 1, we first present two Lemmas.

Lemma 1. *If Ω satisfies AG-GARP, then for all $z^0 \in Z$, $z^0 \in \text{d}CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_A z^0\})$ and $z^0 \in \text{d}CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_S z^0\})$.*

Proof. We prove by contradiction. Assume that $z^0 \notin \text{d}CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_S z^0\})$. Let $C = CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_S z^0\})$, $Z^* = \{z^i \in Z : z^i \tilde{\mathbf{R}}_S z^0\}$ and $\bar{Z} = Z \cap \text{d}C$. W.l.o.g., we relabel indices of $\bar{Z} = \{\bar{z}^i\}_{i=1}^l$ such that $\bar{z}_1^{i+1} < \bar{z}_1^i$ for $i = 1, \dots, l-1$.

The Lemma holds trivially when $Z = \emptyset$. Suppose that $Z \neq \emptyset$. If $z^0 \in \text{int} C$, then there exist $\bar{z}^{i^*} \in \bar{Z}$, and $z^m \in (Z \cap \text{int} C)$ such that $\bar{z}^{i^*} \tilde{\mathbf{R}}_S^0 z^m$. Next we prove the following two claims.

- (i) $\bar{z}^i \tilde{\mathbf{R}}_S^0 \bar{z}^{i+1}$ for $i \in \{1, \dots, l-1\}$;

(ii) $\bar{z}^i \tilde{\mathbf{R}}_S^0 \bar{z}^{i-1}$ for $i \in \{2, \dots, l\}$.

Proof of Claim 1. We start from \bar{z}^1 . Since $\bar{z}^1 \in dC$ and $\bar{z}^1 \tilde{\mathbf{R}}_S \bar{z}^0$, we must have $\bar{z}^1 \tilde{\mathbf{R}}_S^0 \bar{z}^j$ for some $\bar{z}^j \in C$. Then $\bar{z}^j \in \mathbf{B}(r^1)$, which implies $\mathbf{B}(r^1) \cap C \neq \{\bar{z}^1\}$. Also $\bar{z}_1^1 > \bar{z}_1^2$, $\bar{z}^1 \tilde{\mathbf{R}}_S^0 \bar{z}^2$.

Consider \bar{z}^2 . By AG-GARP, we cannot have $\bar{z}^2 \tilde{\mathbf{P}}_S^0 \bar{z}^1$. If $\bar{z}^1 \tilde{\mathbf{R}}_S^0 \bar{z}^2 \tilde{\mathbf{R}}_S^0 \bar{z}^1$, then $\{\bar{z}^1, \bar{z}^2\} \in d\mathbf{B}(r^1)$ and $\mathbf{B}(r^1) = \mathbf{B}(r^2)$. Additionally, $\{\bar{z}^1, \bar{z}^1\} \in dC$, $\mathbf{B}(r^1), \mathbf{B}(r^1)$ are supporting hyperplanes of C . Then both \bar{z}^1 and \bar{z}^2 can only be preferred to other choices in C if both are preferred to \bar{z}^3 and $\bar{z}^3 \in d\mathbf{B}(r^2)$. Since $\bar{z}_1^2 > \bar{z}_1^3$, then $\bar{z}^2 \tilde{\mathbf{R}}^0 \bar{z}^3$. If $\bar{z}^1 \tilde{\mathbf{P}}_S^0 \bar{z}^2$, then $\bar{z}^2 \tilde{\mathbf{R}}_S^0 \bar{z}^3$ holds the same as we derive $\bar{z}^1 \tilde{\mathbf{R}}_S^0 \bar{z}^2$. Therefore, by induction, $\bar{z}^i \tilde{\mathbf{R}}_S^0 \bar{z}^{i+1}$ for $i \in \{1, \dots, l-1\}$.

Proof of Claim 2. We start from \bar{z}^l . Since $\bar{z}^l \in dC$, we must have $\bar{z}^l \tilde{\mathbf{R}}_S^0 \bar{z}^j$ for some $\bar{z}^j \in C$. Because $\bar{z}_1^l < \bar{z}_1^i$ for all $\bar{z}^i \in C$, we cannot have $\bar{z}^l \tilde{\mathbf{R}}_A^0 \bar{z}^j$. Thus, $\bar{z}^l \in \{x_i\}_{i=1}^n$ and $\bar{z}^l \tilde{\mathbf{R}}_A^0 \bar{z}^j$, then $\bar{z}^l \tilde{\mathbf{R}}_A^0 \bar{z}^{l-1}$. Similarly to the proof of Claim 1, we have $\bar{z}^i \tilde{\mathbf{R}}_S^0 \bar{z}^{i-1}$ for $i \in \{2, \dots, l\}$.

- (1) The boundary case: if $i^* = 1$, then $\bar{z}^1 \tilde{\mathbf{R}}_S^0 z^m$. Since $z^m \in \text{int}C$, there is $\bar{z}^k \in \bar{Z}$, such that $z^m \in \text{int}(CMH\{\bar{z}^k\})$, that is $z^m \tilde{\mathbf{P}}_S^0 \bar{z}^2$. Then we have $\bar{z}^1 \tilde{\mathbf{P}}_S^0 \bar{z}^k$. However by claim (i), $\bar{z}^1 \tilde{\mathbf{R}}_S^0 \bar{z}^k$, and we have a contradiction. A symmetric arguments applies for $i^* = l$.
- (2) If $1 < i^* < l$, then if $z_1^m < \bar{z}_1^l$, the contradiction is the same as for the boundary case $i^* = l$; otherwise it is the same as for $i^* = 1$.

This finishes the proof of Lemma 1. □

Lemma 2. Let $C_A = CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_A z^0\})$ and $C_S = CMH(\{z^i \in Z : z^i \tilde{\mathbf{R}}_S z^0\})$. Let $\{z^i\}_{i=1}^l$ be the set of observed choices on dC_A or dC_S such that $z_1^i > z_1^{i+1}$ for all $i = 1, \dots, l-1$. If Ω satisfies AG-GARP then $z^1 \tilde{\mathbf{R}}_A^0 z^2, z^2 \tilde{\mathbf{R}}_A^0 z^3, \dots, z^j \tilde{\mathbf{R}}_A^0 z^0$ and $z^l \tilde{\mathbf{R}}_A^0 z^{l-1}, z^{l-1} \tilde{\mathbf{R}}_A^0 z^{l-2}, \dots, z^{j+1} \tilde{\mathbf{R}}_A^0 z^0$ for some $j \in \{1, \dots, l\}$; and $z^1 \tilde{\mathbf{R}}_A^0 z^2, z^2 \tilde{\mathbf{R}}_A^0 z^3, \dots, z^k \tilde{\mathbf{R}}_A^0 z^0$ and $z^l \tilde{\mathbf{R}}_S^0 z^{l-1} \tilde{\mathbf{R}}_S^0, \dots, \tilde{\mathbf{R}}_S^0 z^{k+1} \tilde{\mathbf{R}}_S^0 z^0$ for some $k \in \{1, \dots, l\}$.

Proof. By Lemma 1, $z^0 \in dC_A$. There is $z^j \in \{z^i\}_{i=1}^l$, such that (z^j, z^0) and (z^0, z^{j+1}) are on the same supporting hyperplane of C respectively. Following the proof of Lemma 1, we have $z^1 \tilde{\mathbf{R}}_A^0 z^2, z^2 \tilde{\mathbf{R}}_A^0 z^3, \dots, z^j \tilde{\mathbf{R}}_A^0 z^0$ and $z^l \tilde{\mathbf{R}}_A^0 z^{l-1}, z^{l-1} \tilde{\mathbf{R}}_A^0 z^{l-2}, \dots, z^{j+1} \tilde{\mathbf{R}}_A^0 z^0$ for some $j \in \{1, \dots, l\}$. □

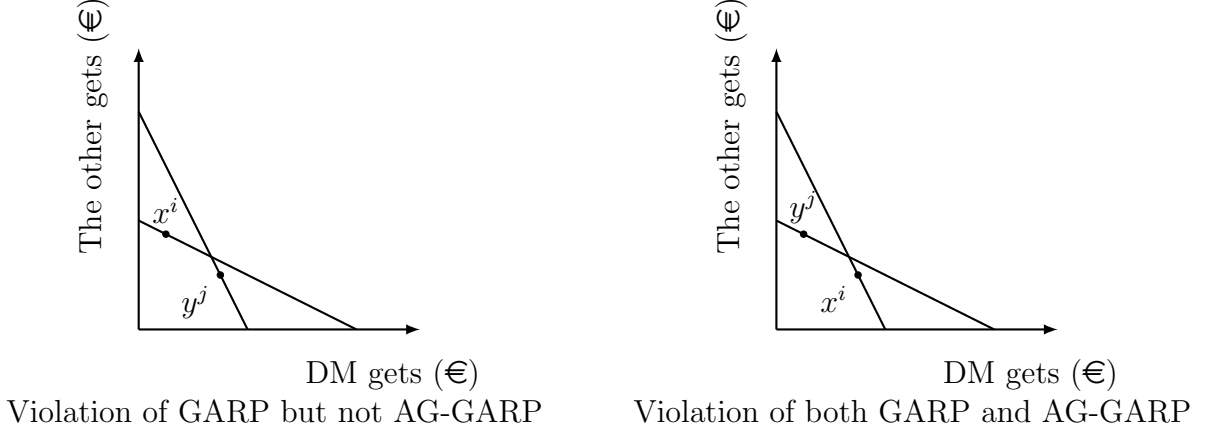


Figure 8: The relation between GARP and AG-GARP, where x^i is chosen in context A and y^i is chosen in context S.

The Agreement axiom only allows one direction of violations of GARP (see Figure 8). Observations in both figures violate GARP. However, if x^i is chosen in context A and y^i is chosen in context S, Agreement allows such choices in the left figure but does not admit those in the right figure. Lemma 1 shows that Agreement is still enough to conclude that observations on the boundary of monotonic convex hull are revealed indifferent. Lemma 2 explains in detail the relation of observations on the boundary of monotonic convex hull. Next we give the proof of Theorem 1.

Proof of Theorem 1. We only prove the sufficiency of AG-GARP to get the representation in our Theorem.

Sketch of the proof. First we construct virtual budgets for every allocation in Ω_S such that these new bundles together with Ω_A satisfy GARP and the extended revealed relation is embedded in the new relation. Then by Proposition 2, we have a continuous, monotonic and quasiconcave altruistic utility rationalising the data. Secondly, based on the constructed altruistic utility we recover the budget for countable dense bundles in \mathbb{R}^2 . Lastly, we add these countable allocations with corresponding virtual budgets to those observations in context S sequentially, and the limit is our desired selfish utility.

Suppose that AG-GARP is satisfied. We show the existence of the selfish utility function. Start by adding allocation x^1 into Ω_S . That is, we construct a virtual budget, p^{1*} , associated with x^1 such that $\Omega_S \cup (x^1, p^{1*})$ satisfies GARP and \tilde{R}_s is embedded in

the revealed relation from $\Omega_S \cup (x^1, p^{1*})$. We discuss the following two cases:

- (1) If $\Omega_S \cup (x^1, p^1)$ satisfies GARP, then we let $p^{1*} = p^1$ and $\Omega_S^1 = \Omega_S \cup (x^1, p^1)$;
- (2) Otherwise, Let $Z^1 = \{y_j\}_{j=1}^m \cup x^1$ and $C = CMH(\{z^i \in Z^0 : z^i \tilde{\mathbf{R}}_S x^1\})$, then $x^1 \in dC$ by Lemma 1. Lemma 2 implies that if $z^i \in Z^1$ and x^1 adjacent on dC , then $z^i \tilde{\mathbf{R}}_S^0 x^1$. AG-GARP implies that not $x^1 \tilde{\mathbf{R}}_S^0 z^i$. Thus $\mathbf{B}(p^1) \cap \text{int}C = \emptyset$. Then $RW(\tilde{\mathbf{R}}_S^0, x^1) \cap C = \emptyset$. Both $RW(\tilde{\mathbf{R}}_S^0, x^1)$ and C are convex, and by the separating hyperplane theorem, the hyperplane separating $RW(\tilde{\mathbf{R}}_S^0, x^1)$ and C has the form of $\{a \in \mathbf{X} : (p_1^1 + \theta^1, p_2^1)(a - x^1) = 0\}$ with some $\theta^1 \geq 0$. Denote $p^{1*} = (p_1^1 + \theta^1, p_2^1)$. Note that for all $x \in \mathbf{X}$, $x^1 \tilde{P}_s x$ implies that $x \in (\tilde{\mathbf{R}}_S^0, x^1)$, that is $x^1 \cdot p^{1*} > x \cdot p^{1*}$. Thus, the new bundle (x^1, p^{1*}) keeps the information from context A.

We repeat this for (x^2, p^2) . We add (x^2, p^{2*}) into Ω_S^1 . By induction we have $\Omega_S^n = \Omega_S \cup \{x^i, p^{i*}\}_{i=1}^n$ and Ω_S^* satisfies GARP. In the same way, we have $\Omega_A^m = \Omega_A \cup \{y^j, q^{j*}\}_{j=1}^m$ and Ω_A^* satisfies GARP. Let $F_0 = \{x^i\}_{i=1}^n \cup \{y^j\}_{j=1}^m$.

Because Ω_A^* and Ω_S^* satisfy GARP, by proposition 2 there exist continuous, strictly increasing and quasiconcave utility functions u and v_0 that rationalise these sets of (virtual) observations. Moreover, because the revealed preference relations on Ω_A^* and Ω_S^* satisfy Agreement by construction, for all $x, y \in F_0$, $u(x) \geq (>) u(y)$ with $x_1 \geq y_1$ implies that $v_0(x) \geq (>) v_0(y)$.

Let $\{\alpha^i\}_{i=1}^\infty$ be the countable rational dense of \mathbb{R}_+^2 , $F_k = \{\alpha^1, \dots, \alpha^k\}$ and $F_\infty = \{\alpha^1, \dots, \alpha^k, \dots\}$.

Lemma 3. *Given F_k , there exist two sets of prices $\{l^i\}_{i=1}^k$ and $\{l'^i\}_{i=1}^k$ such that $\Omega_A^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and $\Omega_S^* \cup \{\alpha^i, l'^i\}_{i=1}^k$ satisfy GARP and jointly satisfy Agreement.*

Proof. We adopt the idea of the proof of Lemma 2 in Reny (2015), based on u , there are virtual prices l_i corresponding to α^i for all i , such that $\Omega_A^* \cup \{\alpha^i, l^i\}_{i=1}^k$ satisfies GARP and u rationalises $\Omega_A^* \cup \{\alpha^i, l^i\}_{i=1}^k$.

Then we show that $\Omega_A^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and Ω_S^* satisfies Agreement. Assume for contradiction that Agreement is violated, then there is some $(\alpha^t, l^t), (\beta, q') \in \Omega_A^*$ and $(\beta, q^*) \in \Omega_S^*$, such that at least one of the following is true:

- (a) $l^t \alpha^t \geq l^t \beta$ and $q^* \beta > q^* \alpha^t$ with $\alpha_1^t \geq \beta_1$;

(b) $l^t \alpha^t > l^t \beta$ and $q^* \beta \geq q^* \alpha^t$ with $\alpha_1^t \geq \beta_1$;

Assume (a) holds, that is, $l^t \alpha^t \geq l^t \beta$ and $q^* \beta > q^* \alpha^t$ with $\alpha_1^t \geq \beta_1$. Since $q^* \beta > q^* \alpha^t$, then $q' \beta > q' \alpha^t$ since $\alpha_1^t \geq \beta_1$ and q^* put more weight on first coordinate than q' . Together with $l^t \alpha^t \geq l^t \beta$, GARP is violated in Ω_A^* , a contradiction. Similarly, we can obtain a contradiction of statement (b). Therefore, $\Omega_A^* \cup \{\alpha^i, l^i\}_{i=1}^k$ and Ω_S^* satisfies Agreement.

Next, we apply the techniques we used before to construct virtual budgets so that there are l'_i corresponding to α_i for all i , so $\Omega_S^* \cup \{\alpha^i, l'_i\}_{i=1}^k$ satisfies GARP, and the construction guarantees that agreement is satisfied. \square

We denote $\Omega_A^k = \Omega_A^* \cup \{\alpha^i, l_i\}_{i=1}^k$ and $\Omega_S^k = \Omega_S^* \cup \{\alpha^i, l'_i\}_{i=1}^k$. Both Ω_A^k and Ω_S^k satisfy GARP, and they jointly satisfy Agreement.

Lemma 4. *There exists a continuous, strictly increasing and quasiconcave v_k that rationalises Ω_S^k such that for all $x, y \in F_0 \cup F_k$ with $x_1 \geq y_1$, $u(x) \geq (>) u(y)$ implies $v_k(x) \geq (>) v_k(y)$, and for all $x, y \in F_0 \cup F_k$ with $x_1 \leq y_1$, $v_k(y) \geq (>) v_k(x)$ implies $u(x) \geq (>) u(y)$*

Proof. By Lemma 3, the virtual budgets $\{\alpha^i, l_i\}_{i=1}^k$ are constructed according to u , thus u rationalises Ω_A^k . Since Ω_S^k satisfies GARP, Proposition 2 implies that there is a continuous, strictly increasing and quasiconcave v_k that rationalises Ω_S^k . The relation of the two utilities can be derived in the same way as before. \square

Define the selfish utility as

$$v := \sup_N \inf_{k \geq N} (v_k),$$

where N is a natural number and the operator works in the following way: given a natural number N , take the infimum of v_k over the set of $\{k \geq N\}$, and then take the supremum over all natural numbers N . Every v_k is a certainty equivalent function, so $\{v_k\}_{k \in N}$ are uniformly bounded and so is v , for all $x \in \mathbf{X}$, $\min\{x_1, x_2\} \leq v(x) \leq \max\{x_1, x_2\}$. We now show some properties of v .

(i) v is continuous.

Since v_k is continuous for all k , v is continuous.

(ii) u and v AG-rationalises Ω .

For all k , altruistic utility u and selfish utility v_k AG-rationalises $\Omega = \Omega_A \cup \Omega_S$, then for very $x, y \in \Omega$, $x \tilde{\mathbf{R}}_s y$ implies that $v_k(x) \geq v_k(y)$. That is, given $x, y \in \Omega$, $x \tilde{\mathbf{R}}_s y$ implies that the inequality

$$v_k(x) \geq v_k(y)$$

holds for all k . Taking the supinf on both side of the equality, we have

$$\sup_N \inf_{k \geq N} (v_k(x)) \geq \sup_N \inf_{k \geq N} (v_k(y)).$$

Therefore, u and $\sup_N \inf_{k \geq N} (v_k)$ AG-rationalises Ω .

(iii) v is quasiconcave .

Given $x, y \in \mathbf{X}$ and $\lambda \in (0, 1)$,

$$\begin{aligned} v_k(\lambda x + (1 - \lambda)y) &\geq \min\{v_k(x), v_k(y)\} \quad \text{holds for all natural } k \\ \Rightarrow \inf_{k \geq N} v_k(\lambda x + (1 - \lambda)y) &\geq \inf_{k \geq N} \min\{v_k(x), v_k(y)\} \\ \Rightarrow \inf_{k \geq N} v_k(\lambda x + (1 - \lambda)y) &\geq \min\left\{\inf_{k \geq N} v_k(x), \inf_{k \geq N} v_k(y)\right\} \\ \Rightarrow \sup_N \inf_{k \geq N} v_k(\lambda x + (1 - \lambda)y) &\geq \sup_N \min\left\{\inf_{k \geq N} v_k(x), \inf_{k \geq N} v_k(y)\right\} \\ \Rightarrow \sup_N \inf_{k \geq N} v_k(\lambda x + (1 - \lambda)y) &\geq \min\left\{\sup_N \inf_{k \geq N} v_k(x), \sup_N \inf_{k \geq N} v_k(y)\right\} \\ \Rightarrow v(\lambda x + (1 - \lambda)y) &\geq \min\{v(x), v(y)\}. \end{aligned}$$

Next we show the following Lemma, which extends Lemma 4 to an infinite number of virtual observations.

Lemma 5. *For all $x, y \in \mathbf{X}$, $u(x) (\geq) > u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>) v(y)$*

Proof. The proof is done in two steps.

(a) First we show for all $x, y \in F_\infty$, $u(x) \geq (>) u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>) v(y)$. Given $x, y \in \{\alpha^i\}_{i=1}^\infty$, $u(x) \geq u(y)$ with $x_1 \geq y_1$, there exists N^* such that for all $N \geq N^*$, $x, y \in F_N$, we have

$$v_N(x) \geq v_N(y) \quad \text{for all } N \geq N^*$$

$$\Rightarrow \inf_{k \geq N} v_k(x) \geq \inf_{k \geq N} v_k(y) \quad \text{for all } N \geq N^*.$$

Naturally, we have

$$\sup_N \inf_{k \geq N} v_{F_k}(x) \geq \sup_N \inf_{k \geq N} v_{F_k}(y), \quad \text{that is, } v(x) \geq v(y).$$

Given $x, y \in \{\alpha^i\}_{i=1}^\infty$, $v(y) \geq v(x)$ with $x_1 \geq y_1$, there exists N^* such that for all $N \geq N^*$, $x, y \in F_N$. Assume for contradiction that there is $K \geq N^*$, $v_K(y) < v_K(x)$, then for all $N \geq K$, $v_N(y) < v_N(x)$, that is $v(y) < v(x)$. Thus, for all $K \geq N^*$, $v_K(y) \geq v_K(x)$. By Lemma 4, $u(y) \geq u(x)$.

Assume for contradiction that there are $x, y \in F_\infty$, such that $u(x) > u(y)$ with $x_1 \geq y_1$ and $v(x) \leq v(y)$. By the previous argument, $v(x) \leq v(y)$ and $x_1 \geq y_1$ implies that $u(y) \geq u(x)$, we have contradiction.

- (b) The relation between u and v holds for F_∞ , so what is left is to show that for all $x, y \in \mathbf{X}$ and not $x, y \in F_\infty$, $u(x) \geq (>) u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>) v(y)$. Given irrational $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$, there exist a rational decreasing sequence $\{a^n\}_{n=1}^\infty$ and increasing sequence $\{b^n\}_{n=1}^\infty$ that satisfy $\lim_{n \rightarrow \infty} a^n = x$ and $\lim_{n \rightarrow \infty} b^n = y$. Then we have $u(a^n) \geq u(x) \geq u(y) \geq u(b^n)$ with $a_1^n \geq b_1^n$ for all n , so $v(a^n) \geq v(b^n)$ for all n . The continuity of v implies

$$v(x) = v(\lim_{n \rightarrow \infty} a^n) = \lim_{n \rightarrow \infty} v(a^n) \geq \lim_{n \rightarrow \infty} v(b^n) = v(\lim_{n \rightarrow \infty} b^n) = v(y).$$

Given irrational $x, y \in \mathbf{X}$, $u(x) > u(y)$ with $x_1 \geq y_1$, there exist a rational increasing sequence $\{a^n\}_{n=1}^\infty$ and decreasing sequence $\{b^n\}_{n=1}^\infty$ that satisfy $u(a^1) > u(b^1)$, $\lim_{n \rightarrow \infty} a^n = x$ and $\lim_{n \rightarrow \infty} b^n = y$. Then we have $u(a^n) > u(b^n)$ with $a_1^n \geq b_1^n$ for all n , so $v(a^n) > v(b^n)$ for all n . The continuity of v implies

$$v(x) = v(\lim_{n \rightarrow \infty} a^n) = \lim_{n \rightarrow \infty} v(a^n) \geq v(a^1) > v(b^1) \geq \lim_{n \rightarrow \infty} v(b^n) = v(\lim_{n \rightarrow \infty} b^n) = v(y).$$

□

Let $x, y \in \mathbf{X}$ with $x > y$ ($x \geq y$ and [either $x_1 > y_1$ or $x_2 > y_2$]), then $u(x) > u(y)$. By Lemma 5, we have $v(x) > v(y)$, that is, v is strictly increasing. Therefore, there exist

continuous, strictly increasing and quasiconcave altruistic utility $u(x)$ and selfish utility $v(x)$ that AG-rationalises Ω . Moreover, for all $x, y \in \mathbf{X}$, $u(x) \geq u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq v(y)$ and for all $x, y \in \mathbf{X}$, $v(x) \geq v(y)$ with $x_1 \leq y_1$ implies that $u(x) \geq u(y)$. This finishes the proof. \square

The following proposition states that two preferences \succsim_1 is MAT \succsim_2 implies that preferences \succsim_1 and \succsim_2 satisfies Agreement.

Proposition 3. *Given two preference relations \succsim_1 and \succsim_2 on \mathbf{X} , represented by differentiable and monotonic utility functions u and v respectively, if \succsim_1 MAT \succsim_2 then \succsim_1 and \succsim_2 satisfy agreement.*

Proof. If \succsim_1 and \succsim_2 satisfy agreement and \succsim_1 is more altruistic, $x \succsim_1 y$ with $x_1 \geq y_1$ implies that $x \succsim_2 y$ for all $x, y \in \mathbf{X}$. Both \succsim_1 and \succsim_2 have utility representations u and v respectively, thus it is equivalent to show that if \succsim_1 MAT \succsim_2 then $u(x) \geq (>)u(y)$ with $x_1 \geq y_1$ implies that $v(x) \geq (>)v(y)$ for all $x, y \in \mathbf{X}$.

Given $x \in \mathbf{X}$, let $f_x : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $f_x(x_1) = x_2$ and $u(a, f_x(a)) = k$ for some constant $k > 0$ for all $a \in \mathbb{R}_+$. That is, f_x denote the indifference curve of preference \succsim_1 through $x \in \mathbf{X}$. Similarly let g_x be the indifference curve of preference \succsim_2 through $x \in \mathbf{X}$. By the definition of MAT, \succsim_1 MAT \succsim_2 implies that $f'_x(a) \geq g'_x(a)$ for all $a \in \mathbb{R}_+$ and $x \in \mathbf{X}$.

Choose any $x, y \in \mathbf{X}$ with $x_1 \geq y_1$ and $u(x) \geq u(y)$. We have $f_x(y_1) \geq y_2$. Define $F_x(a) = f_x(a) - g_x(a)$, then $F'_x(a) = f'_x(a) - g'_x(a) \geq 0$ for all $a \in \mathbb{R}_+$, so F_x is non-decreasing. Moreover, since $F_x(x_1) = f_x(x_1) - g_x(x_1) = x_2 - x_2 = 0$ and $x_1 \geq y_1$, $F_x(y_1) = f_x(y_1) - g_x(y_1) \leq 0$, that is, $f_x(y_1) \leq g_x(y_1)$. Thus we have $g_x(y_1) \geq y_2$, and so $v(x) \geq v(y)$. The proof is similar if we replace the weak inequalities by strict ones. \square

Theorem 3. *Suppose \succsim_A and \succsim_S are represented by CES functions, that is, $u(z) = (\alpha_A z_1^{\rho_A} + (1 - \alpha_A) z_2^{\rho_A})^{1/\rho_A}$ for \succsim_A and $v(z) = (\alpha_S z_1^{\rho_S} + (1 - \alpha_S) z_2^{\rho_S})^{1/\rho_S}$ for \succsim_S . Then Agreement is satisfied if and only if $\alpha_A \leq \alpha_S$ and $\rho_A = \rho_S$.*

Proof. \Leftarrow Assume that $\alpha_A \leq \alpha_S$ and $\rho_A = \rho_S = \rho$. Given $x, y \in \mathbf{X}$, if $x \succsim_A y$ and $x_1 > y_1$, then

$$u(x) = (\alpha_A x_1^\rho + (1 - \alpha_A) x_2^\rho)^{1/\rho} \geq u(y) = (\alpha_A y_1^\rho + (1 - \alpha_A) y_2^\rho)^{1/\rho}.$$

Since $x_1 > y_1$ and $\alpha_A \leq \alpha_S$, then

$$v(x) = (\alpha_S x_1^\rho + (1 - \alpha_S) x_2^\rho)^{1/\rho} \geq v(y) = (\alpha_S y_1^\rho + (1 - \alpha_S) y_2^\rho)^{1/\rho},$$

Thus, we have $x \succsim_S y$.

\Rightarrow Assume that Agreement holds. First we show that Agreement implies that marginal rate of substitution (MRS) of the altruistic preference is smaller than that of selfish preference. Given $\varepsilon > 0$, $\lambda(\varepsilon)$ is the value such that the following indifference holds:

$$(x + \varepsilon, y) \sim_A (x, y + \lambda(\varepsilon)) \quad (3)$$

By Agreement, 3 implies that

$$(x + \varepsilon, y) \succsim_S (x, y + \lambda(\varepsilon)) \quad (4)$$

We write 3 and 4 in utility functions,

$$u(x + \varepsilon, y) = u(x, y + \lambda(\varepsilon)) \Rightarrow v(x + \varepsilon, y) \geq v(x, y + \lambda(\varepsilon)),$$

which is equivalent to,

$$\begin{aligned} \frac{u(x + \varepsilon, y) - u(x, y)}{\varepsilon} &= \frac{u(x, y + \lambda(\varepsilon)) - u(x, y)}{\lambda(\varepsilon)} \\ \Rightarrow \frac{v(x + \varepsilon, y) - v(x, y)}{\varepsilon} &\geq \frac{v(x, y + \lambda(\varepsilon)) - v(x, y)}{\lambda(\varepsilon)}, \end{aligned}$$

That is,

$$\begin{aligned} \frac{\frac{u(x+\varepsilon,y)-u(x,y)}{\varepsilon}}{\frac{u(x,y+\lambda(\varepsilon))-u(x,y)}{\varepsilon}} &= \frac{\varepsilon}{\lambda(\varepsilon)} \\ \Rightarrow \frac{\frac{v(x+\varepsilon,y)-v(x,y)}{\varepsilon}}{\frac{v(x,y+\lambda(\varepsilon))-v(x,y)}{\varepsilon}} &\geq \frac{\varepsilon}{\lambda(\varepsilon)}. \end{aligned}$$

Since u and v are CES functions which are smooth, let $\varepsilon \downarrow 0$ and $K = \lim_{\varepsilon \downarrow 0} \frac{\varepsilon}{\lambda(\varepsilon)}$, then we have

$$\frac{\frac{\partial u}{\partial a_1}}{\frac{\partial u}{\partial a_2}} = MRS_u = K \Rightarrow \frac{\frac{\partial v}{\partial a_1}}{\frac{\partial v}{\partial a_2}} MRS_v \geq K.$$

Therefore, $MRS_v \geq MRS_u$.

Substituting the $MRS_v \geq MRS_u$, with CES functions, we have for all $a = (a_1, a_2) \in \mathbf{X}$,

$$\frac{\alpha_A}{1 - \alpha_A} \left(\frac{a_1}{a_2} \right)^{\rho_A} \geq \frac{\alpha_S}{1 - \alpha_S} \left(\frac{a_1}{a_2} \right)^{\rho_S}$$

Then $a_1 \geq a_2$ and $\rho_A = \rho_S$.

□

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